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On a new approach to meson phenomenology with the Bethe-Salpeter equation

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Abstract We investigate capabilities of the effective interaction in a rainbow-ladder truncated meson model of QCD within a covariant Landau-gauge Bethe-Salpeter-equation approach. Based upon past success for the light- as well as heavy-quark domains, we discuss the range of applicability and features of an effort with comprehensive phenomenological claim and goals.

Keywords meson spectra · bottomonium · Dyson-Schwinger equations · Bethe-Salpeter equation

1 Introduction

The calculation of meson properties in the Dyson-Schwinger-Bethe-Salpeter-equation (DSBSE) approach has enriched the theoretical hadron-physics landscape for many years. In fact, it was realized soon after the conception of the quark picture of hadrons that a relativistic dynamical setup was needed for a more in-depth description of the ever-growing sample of hadron-physics data. Moreover, the phenomenological success of the quark-model hypothesis clearly indicated the convincing potential of a covariant description of hadrons rooted in quantum chromodynamics (QCD), which is now widely accepted to be the theory describing the strong interaction. The modern tools that displayed the capability to achieve this goal are lattice-regularized QCD on one hand and continuum quantum field theoretical methods on the other hand, one of which is the DSBSE approach employed here.

In a phenomenological DSBSE setup one is immediately confronted with the complexity of the infinite, coupled system of QCD's Dyson-Schwinger equations (DSEs) [1]. Thus, the straight-forward rainbow-ladder (RL) truncation of the coupled quark DSE and meson Bethe-Salpeter-equation (BSE) system rose to great popularity quickly [2] after its helpful and QCD-authentic features had been demonstrated. In particular, attention was drawn to relevant Ward-Takahashi identities (WTIs) such as the axial-vector WTI (AVWTI), see, e. g., [3], and its satisfaction in RL truncation (together with the vector WTI [3–10]), which leads to a comprehensively veracious description of the pion and its properties [11]. More precisely, a pion computed from an RL-truncated DSBSE model calculation follows the pattern required by the Goldstone theorem in that it is massless in the chiral limit. For small finite current-quark masses, it follows the well-known Gell-Mann–Oakes–Renner relation; in fact, it was even shown that a generalized version of this relation exists that is valid for all pseudoscalar mesons regardless of their mass and level of excitation [8, 12].

On top of the archetypical treatment of the pion in this approach, it is not surprising that most of the phenomenological studies that followed and which used a sophisticated model interaction in RL truncation focused on the light-quark sector. In addition, reaching larger current quark masses

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and numerically computing bound states of such quarks in a Landau-gauge calculation (for insight in the situation in Coulomb gauge, see, e.g., [13–19]) in Euclidean space like it is used in this approach poses numerical challenges. When these were finally being overcome recently, see, e.g., [20, 21], the model assumptions remained anchored to the light-quark domain nonetheless. While this still gave reasonable results for pseudoscalar and vector mesons, states identified with either radial or orbital angular momentum excitations were not well described [22–30]. While this could be interpreted on a general footing and the conclusion could be drawn that the RL truncation is not sufficient to provide a generally satisfying meson phenomenology and that for such satisfaction to be achieved one needs to include corrections to this truncation or, simply speaking, a quark-gluon vertex more complicated than the bare one, we challenge this line of thinking and attempt a counterexample.

More precisely, we start our version of a phenomenological QCD-model approach via the DSBSE method in the heavy-quark domain. Generalizing on previous accomplishments [31], we allow for more freedom in the effective interaction and test our assumption by comparing our results to the available meson data in the bottomonium system. Herein we present a first look at the possibilities and limitations of the present setup, as well as steps to be taken next to complete this study.

2 Bottomonium in the DSBSE approach

The study of the bottomonium system in the DSBSE approach has been a part of several investigations of meson properties. This section means to put them in perspective with respect to each other and to the comprehensive study to follow up on the present excerpt [32]. In the context of the present setup it is always instructive to note that first simplified attempts at meson spectroscopy including bottomonium were already undertaken several decades ago [33] and later, under certain approximations to the quark propagators that violated the AVWTI, in [34, 35], where also radial excitations were studied. This line of work was continued by investigating corrections beyond RL truncation in a systematic truncation scheme using a simplified model interaction [36, 37].

Separable forms of the BSE kernel were employed mainly to make use of concepts along the lines of heavy-quark effective theory and study heavy-light mesons [38].

In later studies with a full numerical account of the quark propagators and thus an also numerical satisfaction of the AVWTI, heavy quarks were difficult to treat with methods available at the time and so at first efforts focussed on systems involving only light or at most charmed quarks [39]. Bottomonium in this context first appeared only a couple of years ago [40] and soon thereafter several investigations involved bottomonium as an important part for the study of, e.g., effects of the dressing of heavy quarks or various parts of the effective interaction [25, 41–45].

The most recent development regarding bottomonium in this context is given in [31] where bottomonium ground-state masses and decay constants were studied to test the straight-forward applicability of a standard effective interaction to this system simply by adjusting one free model parameter and without subsequent fine-tuning of any of the model parameters, which proved to be successful for all ground states known experimentally.

3 Interaction model and phenomenological setup

In ladder truncation the homogeneous BSE for quark-antiquark bound states reads:

$$\Gamma(p; P) = -C_F \int_q^\Lambda \mathcal{G}((p-q)^2) D_{\mu\nu}^f(p-q) \gamma_\mu S(q_+) \Gamma(q; P) S(q_-) \gamma_\nu, \quad (1)$$

where Γ is the Bethe-Salpeter amplitude (BSA), $C_F = 4/3$ the Casimir color factor, $D_{\mu\nu}^f$ is the free gluon propagator, γ is the Dirac part of the bare quark-gluon vertex, and $\int_q^\Lambda := \int^\Lambda d^4q / (2\pi)^4$ represents a translationally invariant regularization of the integral, with the regularization scale Λ [11]. q and P are the relative and total momenta of the $q\bar{q}$ state, respectively, and the semicolon separates them as four-vector arguments of the BSA. The (anti)quark momenta are $q_+ = q + \eta P$ and $q_- = q - (1 - \eta)P$, where $\eta \in [0, 1]$ is referred to as the momentum partitioning parameter. We use the arbitrariness of the value of η in our covariant framework to set $\eta = 1/2$.

The renormalized dressed quark propagator $S(p)$ is obtained from the corresponding rainbow-truncated quark DSE

$$S(p)^{-1} = (i\gamma \cdot p + m_q) + \Sigma(p), \quad (2)$$

$$\Sigma(p) = C_F \int_q^A \mathcal{G}((p-q)^2) D_{\mu\nu}^f(p-q) \gamma_\mu S(q) \gamma_\nu. \quad (3)$$

$\Sigma(p)$ denotes the quark self-energy, and m_q is the current-quark mass; details of the renormalization of the quark propagator can be found in [11, 46].

The function \mathcal{G} apparent in both Eqs. (1) and (3) is the effective form of the quark-gluon interaction to go with the RL truncated model setup. With $s := (p-q)^2$ we employ the well-established parameterization [46]

$$\frac{\mathcal{G}(s)}{s} = \frac{4\pi^2 D}{\omega^6} s e^{-s/\omega^2} + \frac{4\pi \gamma_m \pi \mathcal{F}(s)}{1/2 \ln[\tau + (1 + s/\Lambda_{\text{QCD}}^2)^2]}. \quad (4)$$

This form has a perturbative limit consistent with the one-loop renormalization group behavior of QCD. While the far infrared is not expected to have a significant impact for our purposes [47], its low and intermediate momentum ranges include some model enhancement to provide the flexibility needed in a phenomenological approach, e. g., to accommodate the correct amount of dynamical chiral symmetry breaking. Furthermore, $\mathcal{F}(s) = [1 - \exp(-s/[4m_t^2])]/s$, $m_t = 0.5$ GeV, $\tau = e^2 - 1$, $N_f = 4$, $\Lambda_{\text{QCD}}^{N_f=4} = 0.234$ GeV, and $\gamma_m = 12/(33 - 2N_f)$ [46].

This model interaction has been used over the past years to successfully describe hadron properties, most prominently but not limited to the ones of pseudoscalar and vector mesons, such as electromagnetic properties [9, 48–52], strong decay widths [53, 54], valence-quark distributions [55, 56], as well as properties at finite temperature [57, 58].

4 Approach

In this section we outline our strategy for obtaining a DSBSE result for the bottomonium spectrum that is most satisfactory in the current setup. Notably, two differences compared to the previous study in [31] appear: First, we attempt to describe the spectrum of not only ground but also radially excited states. Second, we allow additional variation of the model parameters in Eq. (4). Keeping this in mind, we thus test our model effective interaction within the range specified below with regard to the following challenges:

- reproduce the splittings of bottomonium ground-state masses for the states available experimentally for $J = 0, 1, 2$ with the same quality as already achieved in [31]
- in addition, reproduce the splitting of the ground vs. first radially excited state in each channel experimentally available
- alternatively, reproduce the splittings of all first radially excited states with respect to each other, where experimentally available

It is important to note at this point that this is the first study with the declared goal to successfully describe both ground and radially excited meson states in an RL-truncated DSBSE approach. Since it is not clear a priori that such an endeavor can be successful even for the promising realm of heavy-quark bound states, several steps are needed to test model assumptions and restrictions without losing track of where certain changes come from.

The original setup of Maris and Tandy [46] for their interaction was anchored in the light-quark domain and model parameters were adjusted to relevant quantities, namely the pion mass and decay constant as well as the chiral condensate. The relevant term in the effective interaction Eq. (4) is the first one, while the second determines the behavior of calculated results in or towards the perturbative domain. More precisely, the current-quark mass m_q as well as the parameters ω and D were adjusted such that light pseudoscalar and vector meson masses and decay constants were well described by, as it turned out, fixing the product $D \times \omega$ to 0.372 GeV³ and varying ω in the range $[0.3, 0.5]$ GeV. In this way, the choice of $D \times \omega$ and m_q effectively defined a one-parameter model. While the calculated

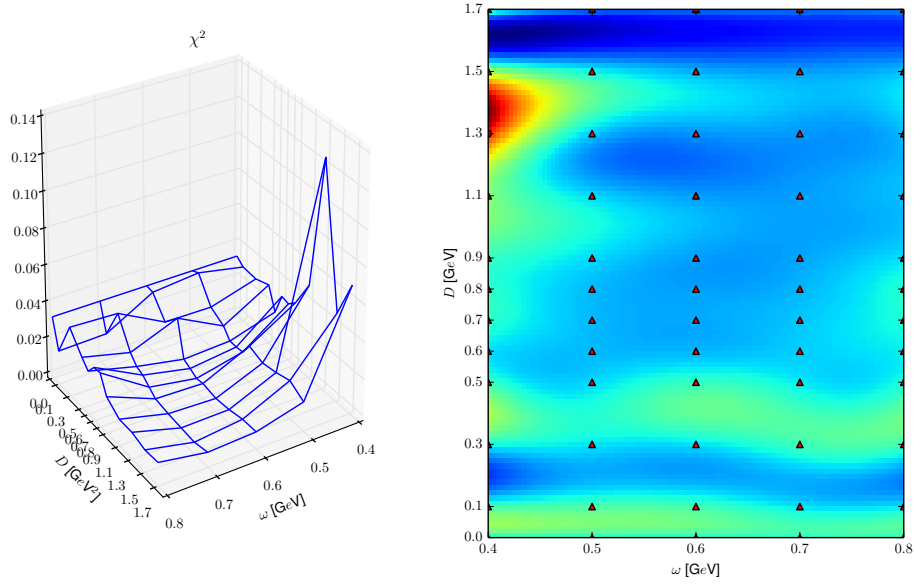


Fig. 1 *Left panel:* χ^2 for a combination of splittings calculated in the $[\omega, D]$ plane for the bottomonium system and compared to experimental data [60]. *Right panel:* contour plot for the same data with interpolated values in between our grid points (red triangles).

pseudoscalar and vector ground-state observables were independent of ω , it was shown later that radial- and orbital-excitation properties strongly depend on ω , even with a fixed value for $D \times \omega$, see [25] and references therein. This is not surprising, since ω corresponds to an inverse range of the intermediate-momentum (i. e., the long-range) part of the effective interaction and one would expect such a parameter to have a noticeable effect on excited but not ground states [59].

In [31] the original value for the product $D \times \omega = 0.372$ was kept and ω fitted to $\omega = 0.61$ GeV to achieve excellent agreement with the experimentally known bottomonium ground states. An equally successful description of radial excitations in addition to the ground states is not possible without allowing both ω and D to vary *independently*, which is what we have done to arrive at the results presented here.

More precisely, as a first step we calculate the mass-splittings among ground and excited states in the bottomonium system for a number of values on an ω - D grid for a fixed value of the bottom current-quark mass and plotted the corresponding χ^2 resulting from our comparison with the available experimental numbers for those splittings, as shown in the left panel of Fig. 1. The right panel of this figure illustrates the behavior of a spline of our grid data.

The second step is then to use the optimal value combination of ω , D on our grid and adjust the bottom current-quark mass m_b such that the experimentally known ground-state masses in the bottomonium system are best reproduced in a least-squares fitting procedure. More concretely, the masses used for this fit have the quantum numbers $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}$, and 2^{++} . The masses of the remaining states are thus predictions of the model.

5 Results and Discussion

The set of various splittings among bottomonium ground and excited states was computed for a bottom-quark mass of $m_b = 3.71$ GeV (given at a renormalization point $\mu = 19$ GeV) and is best reproduced on our grid by the combination $\omega = 0.7$ GeV and $D = 1.3$ GeV². The subsequent least-squares fit of the ground-state masses as described above yields $m_b = 3.635$ GeV; our corresponding results are depicted in Fig. 2, where we also provide the experimental data. The agreement is surprisingly good with the exception of two “extra” states that appear as calculated excitations in the $J^{PC} = 1^{++}$ and

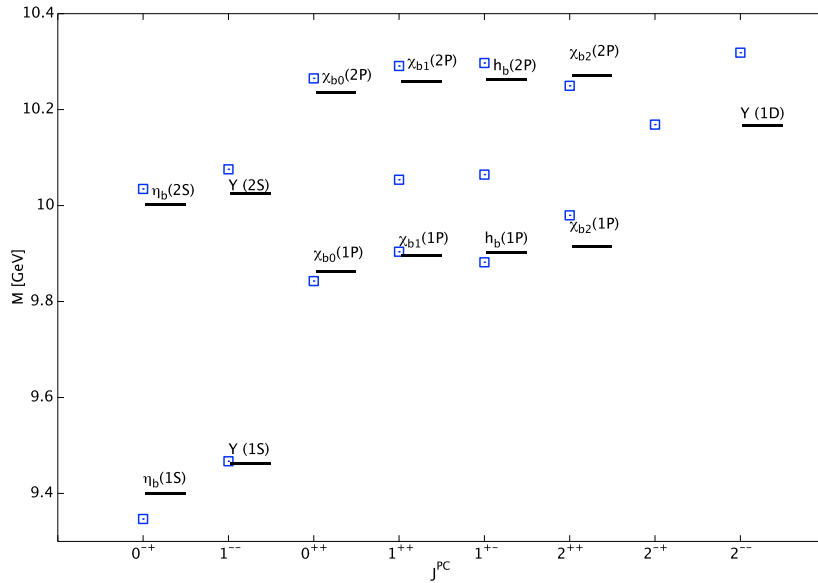


Fig. 2 Bottomonium spectrum: calculated (symbols) versus experimental (lines) data. Error bars are contained inside the symbols for each set of data.

$J^{PC} = 1^{+-}$ channels. Clearly, the nature of these states needs further investigation, which is carried out at the moment. Attempts to include these states as the first radial excitations in their respective channels were unsuccessful, which may also hint at the fact that further degrees of freedom are needed in the effective interaction to provide an overall satisfactory description of the bottomonium system with all its excitations. We note that, since both our numerical as well as experimental uncertainties are smaller than the respective symbol sizes, we have not plotted error bars in Fig. 2.

6 Conclusions

Building on the success of a previous study of the bottomonium ground states in an RL truncated DSBSE approach, we have provided the first successful combined description of ground and radially excited states for the bottomonium system by allowing a wider and more independent variation of the parameters in the effective model interaction. This is the immediate consequence of the idea to anchor the effective quark-gluon interaction in the heavy-quark domain; in addition, our ultimate goal is to provide a comprehensive description of meson spectra along the whole range of quark masses from bottomonium down to the chiral limit, possibly allowing the effective interaction to depend on the current quark mass (see, e.g., [61] for recent insight regarding this topic). This might mimic effects beyond RL truncation such that a successful description of both ground- and excited-state meson properties can be maintained also in the charmonium system and, ultimately, the light-quark sector.

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References

1. Christian S. Fischer. *J. Phys. G*, 32:R253–R291, 2006.
2. Pieter Maris and Craig D. Roberts. *Int. J. Mod. Phys.*, E12:297–365, 2003.
3. H. J. Munczek. *Phys. Rev. D*, 52:4736–4740, 1995.
4. Toshihide Maskawa and Hideo Nakajima. *Prog. Theor. Phys.*, 52:1326–1354, 1974.
5. K.-I. Aoki, et al. *Prog. Theor. Phys.*, 84:683–701, 1990.
6. Taichiro Kugo and Mark G. Mitchard. *Phys. Lett. B*, 282:162–170, 1992.

7. Masako Bando, Masayasu Harada, and Taichiro Kugo. *Prog. Theor. Phys.*, 91:927–948, 1994.
8. Pieter Maris, Craig D. Roberts, and Peter C. Tandy. *Phys. Lett. B*, 420:267–273, 1998.
9. Pieter Maris and Peter C. Tandy. *Phys. Rev. C*, 61:045202, 2000.
10. Pieter Maris and Peter C. Tandy. *Phys. Rev. C*, 62:055204, 2000.
11. Pieter Maris and Craig D. Roberts. *Phys. Rev. C*, 56:3369–3383, 1997.
12. A. Holl, A. Krassnigg, and C. D. Roberts. *Phys. Rev. C*, 70:042203(R), 2004.
13. R. Alkofer, M. Kloker, A. Krassnigg, and R. F. Wagenbrunn. *Phys. Rev. Lett.*, 96:022001, 2006.
14. C. Popovici, P. Watson, and H. Reinhardt. *Phys. Rev. D*, 81:105011, 2010.
15. C. Popovici, P. Watson, and H. Reinhardt. *Phys. Rev.*, D83:125018, 2011.
16. C. Popovici. *Quark sector of Coulomb gauge Quantum Chromodynamics*. PhD thesis, University of Tuebingen, 2011.
17. F. J. Llanes-Estrada, et al. *Fizika B*, 20:63, 2011.
18. S. R. Cotanch and F. J. Llanes-Estrada. *Fizika B*, 20:1, 2011.
19. M. Gomez-Rocha, et al. *Eur. J. Phys. A*, 44:411, 2010.
20. A. Krassnigg. *PoS, Confinement8*:75, 2009.
21. M. Blank and A. Krassnigg. *Comput. Phys. Commun.*, 182:1391, 2011.
22. M. S. Bhagwat, A. Holl, A. Krassnigg, and C. D. Roberts. *Nucl. Phys. A*, 790:10–16, 2007.
23. M. S. Bhagwat, et al., *Few-Body Syst.*, 40:209–235, 2007.
24. C. S. Fischer and R. Williams. *Phys. Rev. Lett.*, 103:122001, 2009.
25. A. Krassnigg. *Phys. Rev. D*, 80:114010, 2009.
26. A. Krassnigg and M. Blank. *Phys. Rev. D*, 83:096006, 2011.
27. M. Blank and A. Krassnigg. *AIP Conf. Proc.*, 1343:349, 2011.
28. S.-X. Qin, L. Chang, Y.-X. Liu, C. D. Roberts, and D. J. Wilson. *Phys. Rev.*, C85:035202, 2012.
29. E. Rojas, B. El-Bennich, and J.P.B.C. de Melo. arXiv:1407.3598.
30. C. S. Fischer, S. Kubrak, and R. Williams. arXiv:1406.4370.
31. M. Blank and A. Krassnigg. *Phys. Rev. D*, 84:096014, 2011.
32. C. Popovici, T. Hilger, M. Gomez-Rocha, and A. Krassnigg. *in preparation*.
33. H. J. Munczek and A. M. Nemirovsky. *Phys. Rev. D*, 28:181, 1983.
34. Herman J. Munczek and Pankaj Jain. *Phys. Rev. D*, 46:438–445, 1992.
35. Pankaj Jain and Herman J. Munczek. *Phys. Rev. D*, 48:5403–5411, 1993.
36. M. S. Bhagwat, et al. *Phys. Rev. C*, 70:035205, 2004.
37. M. Gomez-Rocha, T. Hilger, and A. Krassnigg. arXiv:1408.1077.
38. Mikhail A. Ivanov, Yu. L. Kalinovskiy, and Craig D. Roberts. *Phys. Rev. D*, 60:034018, 1999.
39. A. Krassnigg and P. Maris. *J. Phys. Conf. Ser.*, 9:153–160, 2005.
40. Pieter Maris. *AIP Conf. Proc.*, 892:65–71, 2007.
41. Nicholas Souchlas. arXiv:1006.0942.
42. N. Souchlas and D. Stratakis. *Phys. Rev. D*, 81:114019, 2010.
43. Trang Nguyen, Nicholas A. Souchlas, and Peter C. Tandy. *AIP Conf. Proc.*, 1361:142–151, 2011.
44. Trang Nguyen, Nicholas A. Souchlas, and Peter C. Tandy. *AIP Conf. Proc.*, 1261:13–18, 2010.
45. T. Nguyen, N. A. Souchlas, and P. C. Tandy. *AIP Conf. Proc.*, 1116:327–333, 2009.
46. Pieter Maris and Peter C. Tandy. *Phys. Rev. C*, 60:055214, 1999.
47. Martina Blank, A. Krassnigg, and Axel Maas. *Phys. Rev. D*, 83:034020, 2011.
48. P. Maris and P. C. Tandy. *Nucl. Phys. Proc. Suppl.*, 161:136–152, 2006.
49. A. Holl, A. Krassnigg, P. Maris, C. D. Roberts, and S. V. Wright. *Phys. Rev. C*, 71:065204, 2005.
50. M. S. Bhagwat and P. Maris. *Phys. Rev. C*, 77:025203, 2008.
51. G. Eichmann, A. Krassnigg, M. Schwinzerl, and R. Alkofer. *Annals Phys.*, 323:2505–2553, 2008.
52. Gernot Eichmann. *Phys. Rev. D*, 84:014014, 2011.
53. Dennis Jarecke, Pieter Maris, and Peter C. Tandy. *Phys. Rev. C*, 67:035202, 2003.
54. V. Mader, G. Eichmann, M. Blank, and A. Krassnigg. *Phys. Rev. D*, 84:034012, 2011.
55. T. Nguyen, A. Bashir, C. D. Roberts, and P. C. Tandy. *Phys. Rev.*, C83:062201, 2011.
56. Ian C. Cloet and Craig D. Roberts. *Prog. Part. Nucl. Phys.*, 77:1–69, 2014.
57. P. Maris, Craig D. Roberts, S. M. Schmidt, and P. C. Tandy. *Phys. Rev. C*, 63:025202, 2001.
58. M. Blank and A. Krassnigg. *Phys. Rev. D*, 82:034006, 2010.
59. A. Holl, A. Krassnigg, C. D. Roberts, and S. V. Wright. *Int. J. Mod. Phys. A*, 20:1778–1784, 2005.
60. J. Beringer and others (Particle Data Group). *Phys. Rev.*, D86:010001, 2012.
61. R. Williams. arXiv:1404.2545.